JOURNAL OF MATERIALS SCIENCE **34** (1999) 1219–1227

Influence of experimental parameters on the estimated value of Weibull's modulus

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This paper is devoted to the study of the influence of experimental parameters and calculation hypotheses on the estimated value of Weibull's modulus, *m*, of structural ceramics. Numerical simulation programs have been written and rupture tests have been performed in order to characterize the role of the number of samples tested and thus of the probability estimator. One can thus define an optimal value of the number of samples needed to estimate Weibull's modulus with a given uncertainty. Other numerical programs simulate the effects of the loading rate as well as the effects of Paris' law constant, *A* and the propagation exponent, *n*, on the *m* value.

1. Introduction

The fracture of brittle materials at room temperature is a phenomenon exhibiting random character, the behaviour law between stresses and strains being limited to an elastic zone that is not pre-defined. The statistical Weibull's [1, 2] analysis is one of the means used to overcome this problem.

This analysis supposes an unimodal distribution of flaws governing the ruin and, in the particular case of a solid of volume, V, submitted to an uniform stress field, σ, it expresses the survival probability, P_s, of this volume to the stress, σ, by means of the exponential law

$$P_s(\sigma) = \exp\left[-\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m V\right] \tag{1}$$

known as a "three parameter Weibull's law", where m is Weibull's modulus, σ_u the ultimate stress and σ_0 is a standardization constant.

By convenience, the threshold value σ_u is often considered to be null and Equation 1 becomes a "two parameter Weibull's law" (governed by m and σ_0).

A logarithmic plot of one of the previous laws allows estimation of the value of Weibull's modulus. Indeed, for given values of σ_0 and V, one can write

$$\ln\left(\ln\frac{1}{P_s}\right) = m\ln(\sigma) + Cte \tag{2}$$

One thus obtains the equation of a straight line, the slope of which is equal to Weibull's modulus.

In practice, for a series of tests, a probability estimator, whose most usual expressions are recalled hereafter, is used to allocate a survival probability for each strength level

$$P_s(\sigma) = 1 - i/(N+1)$$
$$P_s(\sigma) = 1 - (i-0.5)/N$$
$$P_s(\sigma) = 1 - (i-0.3)/(N+0.4)$$
$$P_s(\sigma) = 1 - (i-0.375)/(N+0.25)$$

where N is the number of samples, and i is the ith sample of a family ($1 \le i \le N$). Besides this approach, which allows one to estimate Weibull's parameters by a least mean square (LMS) method applied to a cloud of experimental data, one can use the maximum likelihood method [3]. For structural ceramics, the use of this latter method is advocated by the European Standard EN843 [4], which also suggests the form $P_s(\sigma) = 1 - (i-0.5)/N$ for the probability estimator.

Thus, several methods can be used to estimate the m value and it is advisable to determine the confidence that one can have in each of them for given study conditions. The role of the probability estimator, having been studied many times previously [5–8], is not carefully detailed in this paper, which deals chiefly with the influence of experimental parameters, particularly with the number of samples and the loading rate.

2. Discussion

2.1. Influence of the number of samples

Weibull's analysis is a statistical tool devoted to very scattered data, which allows one to predict the behaviour of a large population family from one of its reduced images. In order to use this tool efficiently, it is necessary to validate its stability in regard to the number of samples tested. In others terms, it is good to determine the minimal size of the statistical sampling ensuring an optimized m value. Such a scientific preoccupation also has economical scope because it tends to optimize the number of tests and therefore to reduce their costs.

To reply to this question, we have used an original numerical simulation whose results have been compared with those of experimental measurements.

2.1.1. Numerical simulation

A program composed of two moduli has been developed that allows simulation of fracture tests on lots of samples coming from an ideal family (that is to say, a family that exhibits perfect Weibull's distribution).

The first modulus generates ideal families whose number of samples, average strength and Weibull's modulus are known.

The second modulus simulates, in these families, random sampling of lots smaller than the initial lot. Two types of sampling procedures are possible:

1. Cumulative sampling. Only values unused after constitution of lot i are available for constitution of lot $i+1$. These latter values are added to those of lot i to constitute a larger lot. It is the situation met when, at the end of a series of tests, an experimenter decides to add values to those already obtained. At the limit, all the samples are tested and Weibull's modulus obtained is equal to the modulus of the whole population.
2. Non-cumulative sampling. All the values are available to constitute each lot. One can thus simulate an infinity of sub-lots whose populations are inferior or equal to that of the initial family. This is a situation that would create an infinity of experimenters, each of them making a lot of given population from all the available samples.

Fig. 1a–f illustrates the results of such simulations for ideal initial families having the following characteristics: population, $N = 500, 1000$ and 2000; mean strength, $\bar{\sigma} = 250$ MPA; and Weibull's modulus, $m = 12$.

Extreme values obtained for each type of lot are represented by points, each point corresponding to the arithmetic average of values obtained from ten lots of equal population. The m modulus of each lot is calculated by the maximum likelihood method. One can thus determine the minimal number of samples to test in order to estimate Weibull's modulus within ±10%, for example, of its actual value.

Analysis of all the results so obtained leads to the conclusions shown in the Table I and Fig. 2.

One observes that the number of samples needed to estimate Weibull's modulus within ±10% of its actual value does not increase significantly when the size of the initial family grows (i.e. with regard to the total population, Weibull's modulus decreases in relative value). Consequently, a significant limitation of the number of tests can be considered, by accepting a reasonable relative error in the m value.

2.1.2. Experimental results

Four-point bending fracture tests on ferrite ceramic samples permit constitution of a file of 319 stress values, on which the previous numerical models of sampling (established in the case of an ideal population) have been applied.

2.1.2.1. Weibull's modulus of the initial lot. Weibull's modulus corresponding to the 319 tested samples has been determined: (i) by linear regression according to "two and three parameter" Weibull's laws using the four forms (recalled in Section 1) for the probability estimator; and (ii) by the maximum likelihood method. The results thus obtained are gathered in Table II, and show that:

TABLE I Number of samples needed to obtain m within ±10% of its actual value

	Family population									
Sampling	50	100	200	300	400	500	750	1000	1500	2000
Non-cumulative	29	55	65	102	115	140	155	180	220	220
Cumulative	23	42	60	65	100	135	140	150	200	200

TABLE II Dependance of m on the probability estimator and on the calculation method

Methods	Estimator 1 $1 - i/(N+1)$	Estimator 2 $1 - (i-0.5)/N$	Estimator 3 $1 - (i-0.3)/(N+0.4)$	Estimator 4 $1 - (i-0.37)/(N+0.25)$
LMS (three parameters)	$m = 10.5$	$m = 9.47$	$m = 9.94$	$m = 9.77$
$\sigma_{moy} = 133.5$[a]	$r = 0.983$	$r = 0.985$	$r = 0.984$	$r = 0.984$
	$\sigma_{min} = 81$[a]	$\sigma_{min} = 81$[a]	$\sigma_{min} = 81$[a]	$\sigma_{min} = 81$[a]
	$\sigma_u = 20$[a]	$\sigma_u = 32.4$[a]	$\sigma_u = 26.8$[a]	$\sigma_u = 28.8$[a]
	$\sigma_0 = 30.5$	$\sigma_0 = 23.5$	$\sigma_0 = 26.6$	$\sigma_0 = 25.5$
LMS (two parameters)	$m = 12.44$	$m = 12.72$	$m = 12.59$	$m = 12.64$
$\sigma_u = 0$[a]	$r = 0.982$	$r = 0.984$	$r = 0.983$	$r = 0.983$
	$\sigma_0 = 43.9$	$\sigma_0 = 45$	$\sigma_0 = 44.5$	$\sigma_0 = 44.7$
Maximum likelihood		$m = 11.32$	$\sigma_{moy} = 139.97$[a]	

[a]Values are in megapascals.

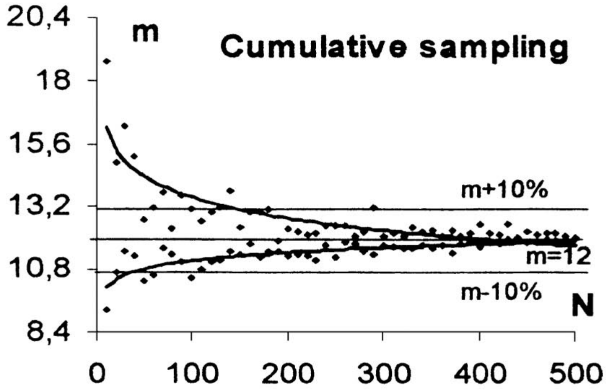

Figure 1a

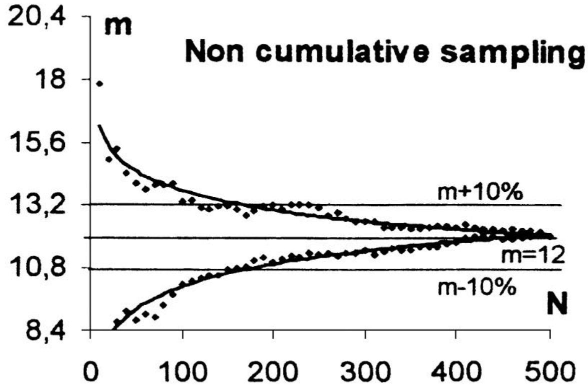

Figure1d

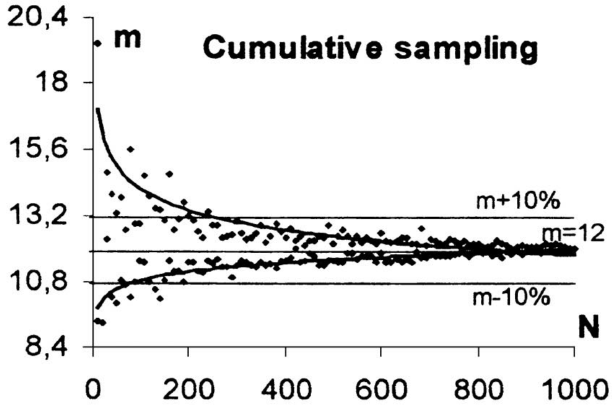

Figure 1b

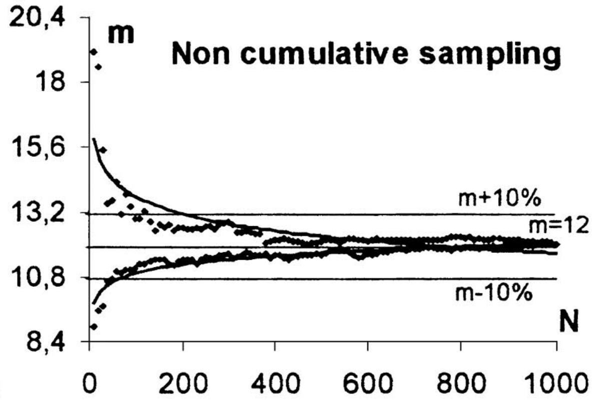

Figure 1e

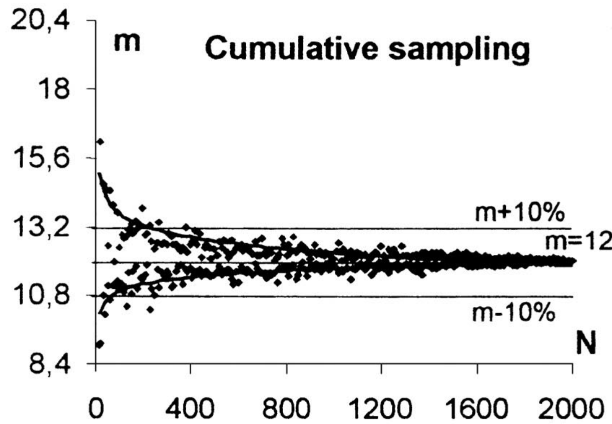

Figure 1c

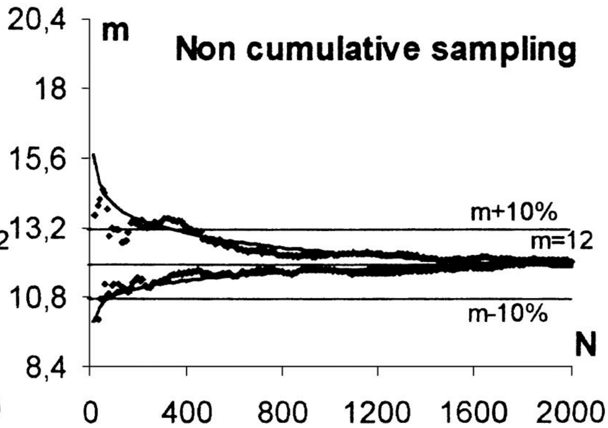

Figure 1f

Figure 1 Weibull's modulus versus *N* (number of samples) for lots obtained by cumulative and non-cumulative sampling: (♦♦♦) real values; (—) tendency curve.

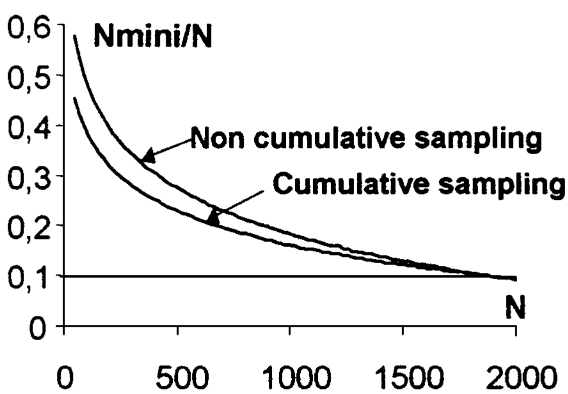

Figure 2 Relative number of samples needed to obtain *m* within ±10% of its actual value.

1. Whatever the probability estimator, the "three parameter" Weibull's law gives the weakest *m* values and the "two parameter" law gives the highest values.

2. The maximum likelihood method leads to values ranging between the precedents, but which are far closer to raised values than to weak values.

2.1.2.2. Application of the model to experimental results. From the 319 previous real values, lots have been constituted by cumulative and non-cumulative sampling according to a process similar to that used for ideal families.

Fig. 3a and b illustrates the results obtained and shows that:

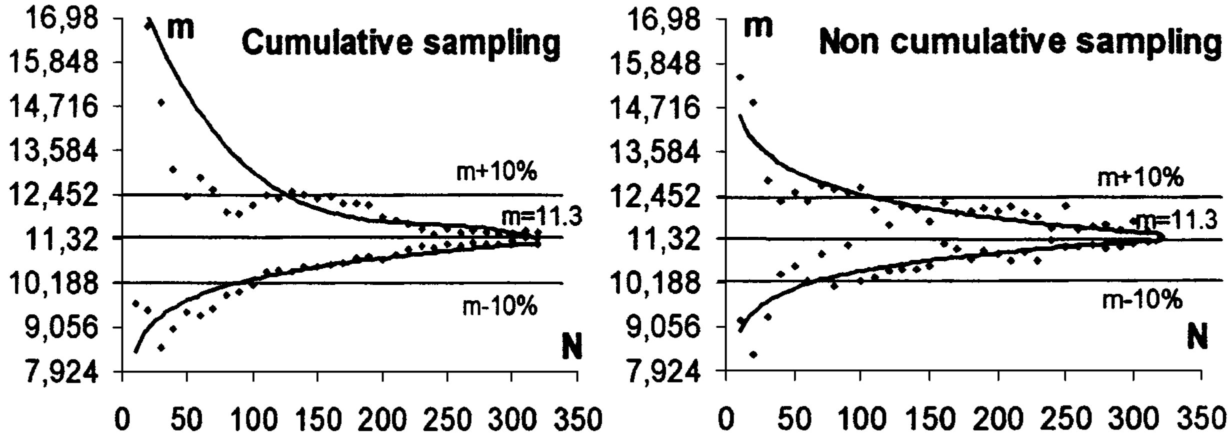

Figure 3a   Figure 3b

Figure 3 Weibull's modulus versus N for experimental data: (♦♦♦) real values; (—) tendency curve.

1. Whatever the considered method, 100 samples are necessary to estimate Weibull's modulus within ±10% of its real value.
2. This conclusion agrees well with predictions of the simulation (cf. Fig. 1a–f) and therefore validates this latter.

2.1.2.3. Remark on the maximum likelihood method. Figs 4 and 5 are "two and three parameter" Weibull's plots of the initial family (probability estimator, $P_s(\sigma) = 1 - (i - 0.5)/N$). One observes the presence of five stress values located to the left extremity of the distribution and obviously out of the average tendency.

Tables III–VII correspond to files containing between 314 and 318 values, obtained by eliminating each time the weakest stress (left extremity) of the preceding file, until the five values, apparently out of the distribution, have been eliminated.

One observes that progressive suppression of data out of the distribution:

1. Decreases strongly (−300%) the results of the "three parameter" model.
2. Increases 10–15% of the results of the "two parameter" model.
3. Increases very slightly (3.5%) the results of the maximum likelihood method, which thus appears as less dependent on "suspicious" values.

TABLE III 318 samples

Methods	Estimator 1 $1 - i/(N+1)$	Estimator 2 $1 - (i - 0.5)/N$	Estimator 3 $1 - (i - 0.3)/(N + 0.4)$	Estimator 4 $1 - (i - 0.37)/(N + 0.25)$
LMS (three parameters)	$m = 7.22$	$m = 6.78$	$m = 6.97$	$m = 6.9$
$\sigma_{moy} = 133.7^a$	$r = 0.986$	$r = 0.988$	$r = 0.987$	$r = 0.987$
	$\sigma_{min} = 87.2^a$	$\sigma_{min} = 87.2^a$	$\sigma_{min} = 87.2^a$	$\sigma_{min} = 87.2^a$
	$\sigma_u = 57.2^a$	$\sigma_u = 62.8^a$	$\sigma_u = 60.4^a$	$\sigma_u = 61.2^a$
	$\sigma_0 = 11.3$	$\sigma_0 = 9.27$	$\sigma_0 = 10.12$	$\sigma_0 = 9.84$
LMS (two parameters)	$m = 12.94$	$m = 13.21$	$m = 13$	$m = 13.136$
$\sigma_u = 0^a$	$r = 0.982$	$r = 0.9818$	$r = 0.982$	$r = 0.982\,04$
	$\sigma_0 = 45.9$	$\sigma_0 = 46.9$	$\sigma_0 = 46.5$	$\sigma_0 = 44.7$
Maximum likelihood		$m = 11.43$	$\sigma_{moy} = 140.07^a$	

[a]Values are in megapascals.

TABLE IV 317 samples

Methods	Estimator 1 $1 - i/(N+1)$	Estimator 2 $1 - (i - 0.5)/N$	Estimator 3 $1 - (i - 0.3)/(N + 0.4)$	Estimator 4 $1 - (i - 0.37)/(N + 0.25)$
LMS (three parameters)	$m = 4.59$	$m = 4.41$	$m = 4.46$	$m = 4.43$
$\sigma_{moy} = 134^a$	$r = 0.993$	$r = 0.994$	$r = 0.993$	$r = 0.993$
	$\sigma_{min} = 98.3^a$	$\sigma_{min} = 98.3^a$	$\sigma_{min} = 98.3^a$	$\sigma_{min} = 98.3^a$
	$\sigma_u = 85.2^a$	$\sigma_u = 87.6^a$	$\sigma_u = 86.8^a$	$\sigma_u = 87.2^a$
	$\sigma_0 = 2.42$	$\sigma_0 = 2$	$\sigma_0 = 2.2$	$\sigma_0 = 2$
LMS (two parameters)	$m = 13.3$	$m = 13.55$	$m = 13.44$	$m = 13.48$
$\sigma_u = 0^a$	$r = 0.978$	$r = 0.975$	$r = 0.977$	$r = 0.976$
	$\sigma_0 = 47.4$	$\sigma_0 = 48.3$	$\sigma_0 = 47.9$	$\sigma_0 = 48$
Maximum likelihood		$m = 11.53$	$\sigma_{moy} = 140.15^a$	

[a]Values are in megapascals.

TABLE V 316 samples

Methods	Estimator 1 $1-i/(N+1)$	Estimator 2 $1-(i-0.5)/N$	Estimator 3 $1-(i-0.3)/(N+0.4)$	Estimator 4 $1-(i-0.37)/(N+0.25)$
LMS (three parameters)	$m=4$	$m=3.87$	$m=3.93$	$m=3.89$
$\sigma_{moy}=134$[a]	$r=0.994$	$r=0.995$	$r=0.995$	$r=0.995$
	$\sigma_{min}=100$[a]	$\sigma_{min}=100$[a]	$\sigma_{min}=100$[a]	$\sigma_{min}=100$[a]
	$\sigma_u=91.6$[a]	$\sigma_u=93.6$[a]	$\sigma_u=92.8$[a]	$\sigma_u=93.2$[a]
	$\sigma_0=1.37$	$\sigma_0=1.14$	$\sigma_0=1.23$	$\sigma_0=1.17$
LMS (two parameters)	$m=13.47$	$m=13.72$	$m=13.61$	$m=13.65$
$\sigma_u=0$[a]	$r=0.974$	$r=0.971$	$r=0.973$	$r=0.972$
	$\sigma_0=48$	$\sigma_0=48.9$	$\sigma_0=48.5$	$\sigma_0=48.7$
Maximum likelihood		$m=11.6$	$\sigma_{moy}=140.23$[a]	

[a]Values are in megapascals.

TABLE VI 315 samples

Methods	Estimator 1 $1-i/(N+1)$	Estimator 2 $1-(i-0.5)/N$	Estimator 3 $1-(i-0.3)/(N+0.4)$	Estimator 4 $1-(i-0.37)/(N+0.25)$
LMS (three parameters)	$m=3.26$	$m=3.17$	$m=3.2$	$m=3.21$
$\sigma_{moy}=134.2$[a]	$r=0.996$	$r=0.997$	$r=0.997$	$r=0.997$
	$\sigma_{min}=105$[a]	$\sigma_{min}=105$[a]	$\sigma_{min}=105$[a]	$\sigma_{min}=105$[a]
	$\sigma_u=99.2$[a]	$\sigma_u=100.4$[a]	$\sigma_u=100$[a]	$\sigma_u=100$[a]
	$\sigma_0=0.52$	$\sigma_0=0.44$	$\sigma_0=0.45$	$\sigma_0=0.47$
LMS (two parameters)	$m=13.61$	$m=13.85$	$m=13.75$	$m=13.78$
$\sigma_u=0$[a]	$r=0.97$	$r=0.966$	$r=0.968$	$r=0.967$
	$\sigma_0=48.54$	$\sigma_0=49.45$	$\sigma_0=49$	$\sigma_0=49.2$
Maximum likelihood		$m=11.66$	$\sigma_{moy}=140.3$[a]	

[a]Values are in megapascals.

TABLE VII 314 samples

Methods	Estimator 1 $1-i/(N+1)$	Estimator 2 $1-(i-0.5)/N$	Estimator 3 $1-(i-0.3)/(N+0.4)$	Estimator 4 $1-(i-0.37)/(N+0.25)$
LMS (three parameters)	$m=2.9$	$m=2.79$	$m=2.83$	$m=2.84$
$\sigma_{moy}=134.3$[a]	$r=0.997$	$r=0.996$	$r=0.996$	$r=0.996$
	$\sigma_{min}=108$[a]	$\sigma_{min}=108$[a]	$\sigma_{min}=108$[a]	$\sigma_{min}=108$[a]
	$\sigma_u=102$[a]	$\sigma_u=104$[a]	$\sigma_u=103.6$[a]	$\sigma_u=103.6$[a]
	$\sigma_0=0.27$	$\sigma_0=0.22$	$\sigma_0=0.23$	$\sigma_0=0.24$
LMS (two parameters)	$m=13.71$	$m=13.94$	$m=13.84$	$m=13.87$
$\sigma_u=0$[a]	$r=0.967$	$r=0.962$	$r=0.964$	$r=0.964$
	$\sigma_0=48.92$	$\sigma_0=49.75$	$\sigma_0=49.4$	$\sigma_0=0.2249.52$
Maximum likelihood		$m=11.72$	$\sigma_{moy}=140.36$[a]	

[a]Values are in megapascals.

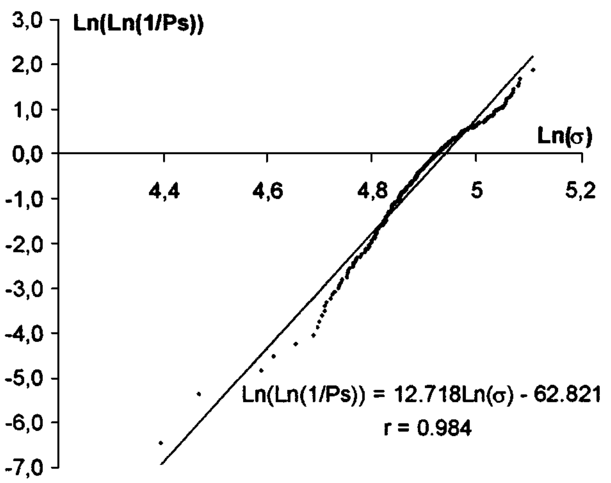

Figure 4 LMS Weibull's plot with $\sigma_u=0$ MPa (two parameters).

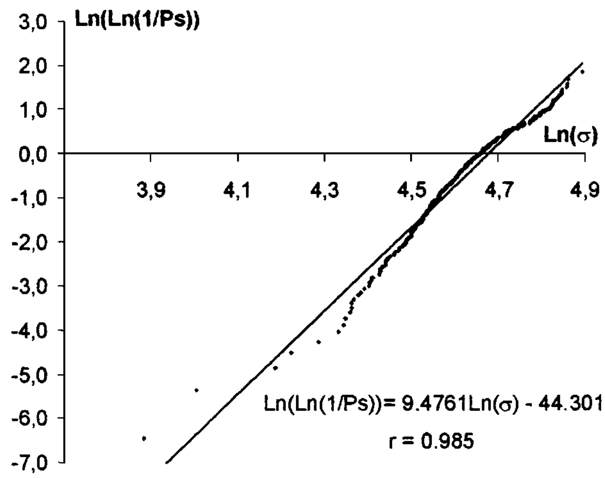

Figure 5 LMS Weibull's plot with $\sigma_u=20$ MPa (three parameters).

2.2. Influence of loading rate

2.2.1. Introduction

The rupture of structural ceramics [9] starts in mode I on the flaw whose stress intensity factor first reaches the critical value, K_{Ic}.

The stress intensity factor in opening mode is given by the relationship

$$K_{\mathrm{I}} = Y\sigma_{\mathrm{a}}a^{1/2} \tag{3}$$

in which Y is a shape factor, σ_a the applied stress and a the flaw size.

Formally, K_{I} can reach the critical value K_{Ic}:

1. By an increase of flaw size under constant applied stress (one speaks then of "critical flaw size").
2. By an increase of applied stress, whereas the flaw size remains constant (one speaks then of "critical stress").

But, in fact, simultaneous increase of these two parameters leads to ruin.

In stage 1, the crack velocity, V, is given by Paris' law

$$V = \frac{\mathrm{d}a}{\mathrm{d}t} = AK_{\mathrm{I}}^{n} \tag{4}$$

where A and n (propagation exponent) are experimental constants, and t is the time.

For high loading rate values, rupture is chiefly governed by the rapid increase of applied stress, the size of the flaw remaining nearly constant (inert rupture): values of strength thus determined lead therefore to the "true value" of Weibull's modulus. But, in fact, tests are performed with relatively weak loading rates (typically 60 MPa s^{-1}) so, when rupture occurs, flaws have increased their size by a slow crack growth mechanism.

The problem is therefore to determine the influence of this slow propagation on the Weibull parameters of the initial distribution.

To reply to this question, we have written a program (cf. the flow chart in Fig. 6) that simulates in an iterative manner the growth of a flaw of known initial size within a material whose entire rupture properties are also known. Each time, one calculates the instantaneous value of the stress intensity factor, the value of the crack velocity and, finally, the resulting increase in flaw size is determined. The process is reiterated until the K_{I} value reaches the material toughness. In the last stage, one builds a stress file of the tested samples, from which one deduces Weibull's modulus at the loading rate considered.

2.2.2. Hypotheses

An ideal initial family of 200 samples has been constituted on the basis of the mechanical characteristics obtained in a previous work [10] dealing with a commercial quality alumina (Degussa AL23):

$$m = 12; n = 22; \ln(A) = -340; K_{\mathrm{Ic}} = 3.7\,\mathrm{MPa\,m}^{-1/2}$$

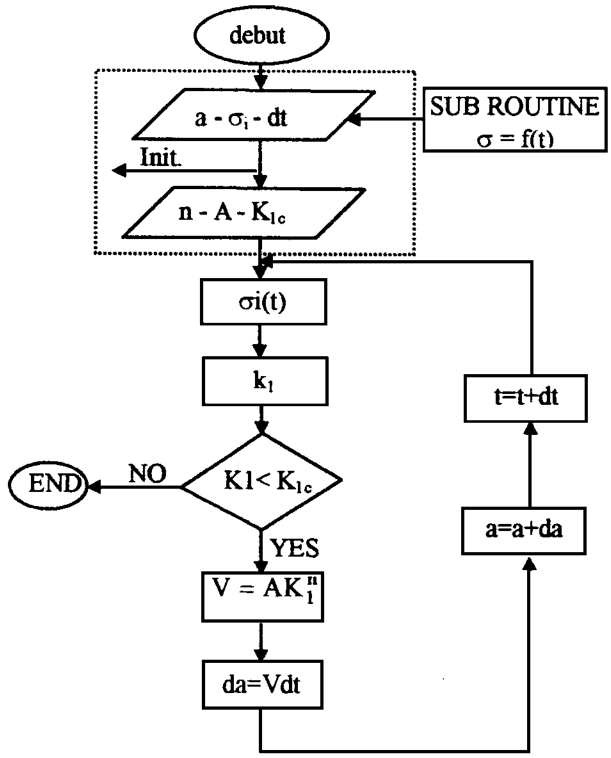

Figure 6 Block diagram of the slow crack growth simulation program.

Because of the weakness of the relative flaw size, the shape factor, Y, is taken equal to 1.9 and the probability estimator, P_{s}, is always $1-(i-0.5/N)$.

The constant rate in stage 2 is fixed [10] to $10^{-4}\,\mathrm{m\,s}^{-1}$ and the propagation in stage 3 is neglected. Loading rates can vary from 0.2 and 200 000 MPa s^{-1}.

2.2.3. Results

Fig. 7 illustrates the evolution of Weibull's plot of the previous family when the stress rate varies from 2 to 4600 MPa s^{-1}. One observes that, whatever the loading rate, $\ln[\ln(1/P_{\mathrm{s}})]$ versus $\ln(\sigma)$ is always represented by a straight line, the slope of which (cf. Weibull's modulus) decreases when the stress rate increases.

In the scanned range, m decreases from 13.9 to 12.2 (-11%), while the average strength grows at the same time from 188 to 249.6 MPa ($+30\%$).

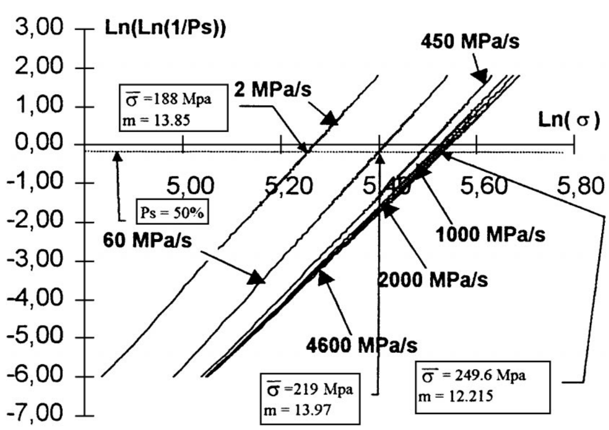

Figure 7 Variation of Weibull's plot with the loading rate.

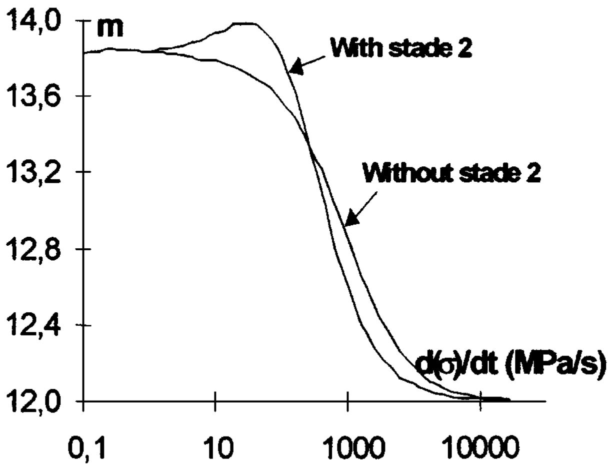

Figure 8 Weibull's modulus versus $\partial\sigma/\partial t$.

Finally, Fig. 8 shows the continuous variation of Weibull's modulus according to stress rate, when stage 2 in the curve $V(K)$ is or is not taken into account.

Figs 7 and 8 show also that:

1. The sensitivity of Weibull's modulus to the stress rate is at a maximum in the range 2–2000 MPa s^{-1}, which corresponds to usual loading rates.
2. Stage 2 does not appear to influence the life time, but seems to influence variations in Weibull's modulus with the stress rate.

2.2.4. Influence of Paris' law parameters on Weibull's modulus

Parameters A and n of Paris' law are determined experimentally, classically by tests of double torsion [8, 9, 11] or dynamic fatigue at constant rates [12, 13]. The results obtained by using these two methods are rarely in good agreement and the important discrepancies frequently observed render hazardous the life-time prediction for components in use. We have focused our attention on the influence of these parameters on Weibull's modulus. By using the numerical program described in the previous paragraph, we have characterized the variations of Weibull's modulus according to the stress rate for different A and n values ($\ln(A) \in [-350; -330]$); ($n \in [21; 23]$). The studied family is composed of 200 samples and the shape factor, Y, is always fixed to 1.9.

The other material characteristics are: $m = 12$; $K_{Ic} = 3.7\,\text{MPa}\,\text{m}^{-1/2}$. The probability estimator remains unchanged.

In order to simplify the reading and interpretation of the numerous results thus obtained, we have used a tridimensional plot in which Weibull's modulus is represented according to:

1. $\partial\sigma/\partial t$ and n for a given value of $\ln(A)$.
2. $\ln(A)$ and $\partial\sigma/\partial t$ for a given value of n.

These results take into account the propagation in mode 2 with $V = 10^{-4}\,\text{m}\,\text{s}^{-1}$.

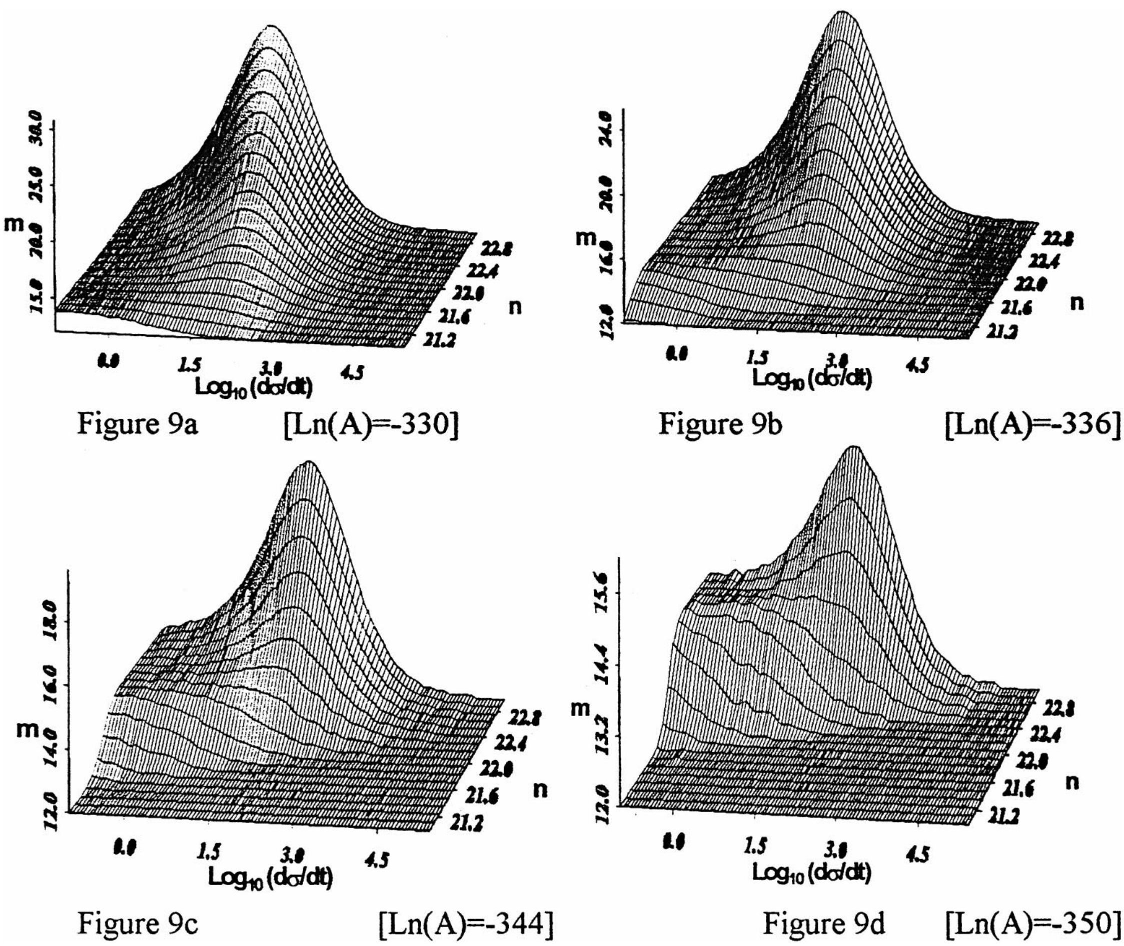

Figure 9 Weibull's modulus versus $\partial\sigma/\partial t$ and n for $-330 \le \ln(A) \le -350$.

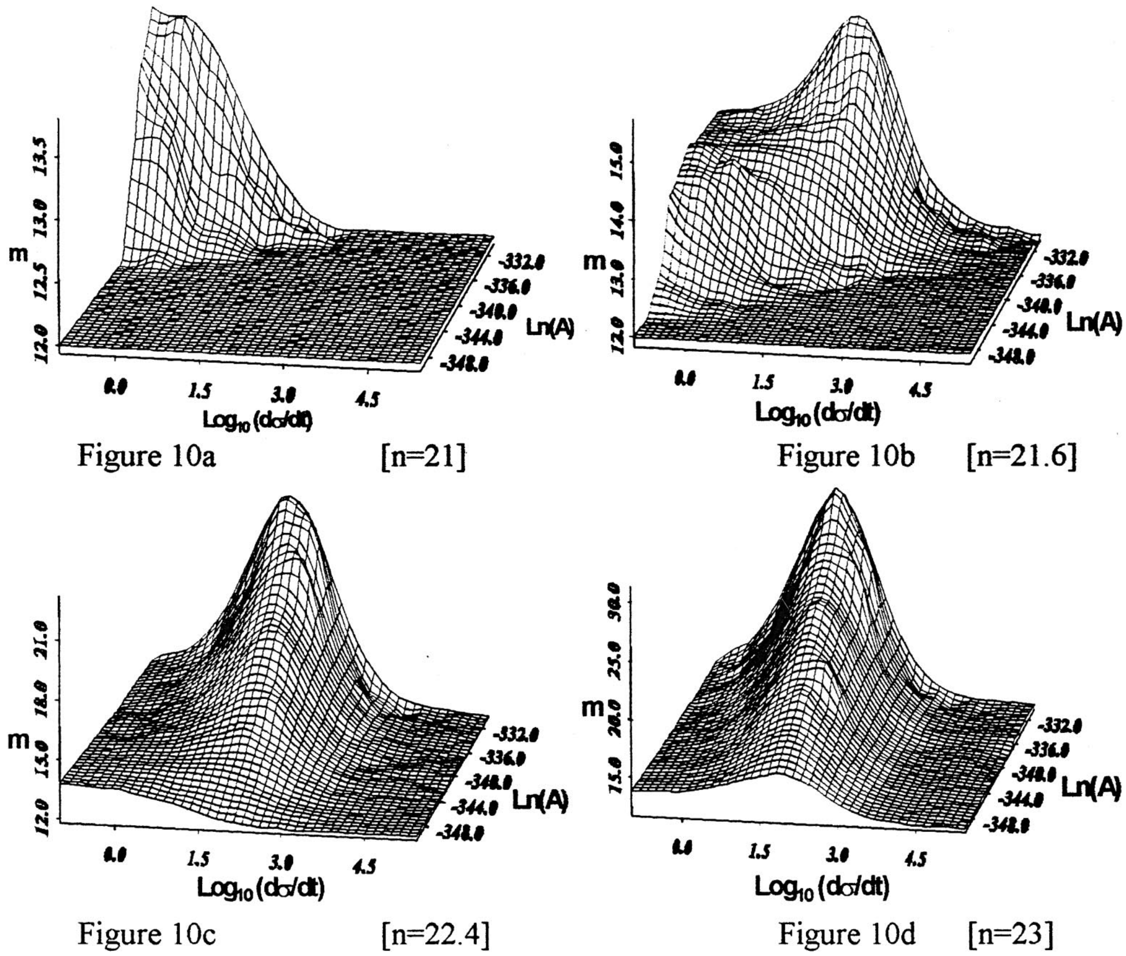

Figure 10 Weibull's modulus versus $\partial\sigma/\partial t$ and ln(A) for $21 \leq n \leq 23$.

2.2.4.1. Ln(A) *is fixed;* $\partial\sigma/\partial$t *and* n *vary.* Fig. 9a–d illustrates the results obtained for $n \in [21; 23]$. One observes that:

1. For a given stress rate, Weibull's modulus exhibits significant variations when the parameters A and n vary.
2. The curves reach a maximum for $n = 23$ and for a stress rate value lying in the range generally used by experimenters (50–2000 MPa s^{-1}). One also observes that when A decreases, the maximum value reached by m also decreases ($15 < m_{max} < 31$), whereas the correspondent stress rate increases.
3. The m value is slightly sensitive to the parameters of simulation if: (i) the stress rate is high ($\partial\sigma/\partial t > 2000$ MPa s^{-1}), whatever the A value is; (ii) the stress rate and the n value are weak (these conditions are satisfied in ranges whose extent increases when the A value decreases).

2.2.4.2. n *is fixed;* $\partial\sigma/\partial$t *and ln*(A) *vary.* Fig. 10a–d illustrates results obtained for ln(A) $\in [-350; -330]$. One observes that:

1. In the range $\partial\sigma/\partial t \in [50; 2000$ MPa $s^{-1}]$ a maximum appears, which corresponds the greatest A value, whose value increases with that of n.
2. The m value is slightly sensitive to the parameters of simulation if: (i) the stress rate is high ($\partial\sigma/\partial t > 2000$ MPa s^{-1}), whatever the ln(A) value is; (ii) the stress rate and the A value are weak (these conditions are satisfied in ranges whose extent increases when the n value increases).

Remark: when stage 2 of the $V(K)$ curve is neglected, the results present important irregularities due to numerical instabilities. This can be explained by very delicate numerical control of the crack propagation from this stage.

3. Conclusions

Although it is generally considered as a characteristic material parameter, Weibull's modulus appears sensitive to experimental conditions implemented for its determination.

For an unimodal distribution of flaws governing ruin, the estimated m value depends indeed, on the form of the probability estimator, on the number of samples tested, on the loading rate and on the value of the A and n parameters of Paris' law.

The present work shows, chiefly, that:

1. The maximum likelihood method, recommended by the European Standards, gives results more reliable than least mean square methods.
2. For a given material, a minimum of 180 samples is needed to estimate the m value within ±10% of its real value.

3. In the range of loading rates generally used (2–2000 MPa s^{-1}) Weibull's modulus can vary from about 16% as compared with its real value.

4. For some values of the couple (A, n) of Paris' law, Weibull's modulus exceeds 50% of its real value.

In conclusion, the numerals given beyond the decimal point are never significant and, from a practical standpoint, it would be good to match the values given for Weibull's modulus of their estimated uncertainties.

Acknowledgements

The authors thank Dr T. Jaccon and Dr J. J. Le Gouic Martun for their help for the supply of strength data.

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Received 27 July
and accepted 26 August 1998